

Written Exam Economics summer 2016

**Monetary Policy**

June 8

(3-hour closed book exam)

Please note that the language used in your exam paper must correspond to the language of the title for which you registered during exam registration. I.e. if you registered for the English title of the course, you must write your exam paper in English. Likewise, if you registered for the Danish title of the course or if you registered for the English title which was followed by “eksamen på dansk” in brackets, you must write your exam paper in Danish.

**This exam question consists of 4 pages in total**

Questions 1, 2 and 3 each weigh 1/3. These weights, however, are only indicative for the overall evaluation.

### QUESTION 1:

Evaluate whether the following statements are true or false. Explain your answers.

- (i) In the Lucas Island model with imperfect information, higher “local” monetary volatility, relative to “aggregate” volatility, makes the real effect of aggregate money shocks smaller.
- (ii) By the logic of the Poole (1970) model, the United States’ Federal Reserve normally uses the nominal interest rate as the policy instrument because the volatility of money-market shocks are negligible.
- (iii) The optimal inflation rate in the simple New-Keynesian model with sticky goods prices is given by the Friedman rule.

### QUESTION 2:

Consider an infinite-horizon economy in discrete time, where utility of the representative agent is given by

$$U = \sum_{t=0}^{\infty} \beta^t u(c_t, n_t), \quad 0 < \beta < 1, \quad (1)$$

with

$$u(c_t, n_t) \equiv \log c_t - \frac{1}{1+\eta} n_t^{1+\eta}, \quad \eta > 0,$$

where  $c_t$  is consumption and  $n_t$  is employment. Agents face the budget constraint

$$c_t + m_t + b_t = f(n_t) + \tau_t + \frac{1+i_{t-1}}{1+\pi_t} b_{t-1} + \frac{1}{1+\pi_t} m_{t-1}, \quad (2)$$

where  $m_t$  is real money balances at the end of period  $t$ ,  $b_t$  is real bond holdings,  $\tau_t$  denotes real monetary transfers from the government,  $i_t$  is the nominal interest rate on bonds, and  $\pi_t$  is the inflation rate. Function  $f$  is defined as

$$f(n_t) \equiv A n_t^{1-\alpha}, \quad A > 0, \quad 1 > \alpha > 0.$$

Agents also face a cash-in-advance constraint

$$c_t \leq \frac{1}{1+\pi_t} m_{t-1} + \tau_t. \quad (3)$$

- (i) Derive the relevant conditions for optimal behavior of the representative agent. For this purpose, set up the value function

$$V(m_{t-1}, b_{t-1}) = \max_{c_t, n_t, m_t} \left\{ u(c_t, n_t) + \beta V(m_t, b_t) - \mu_t \left( c_t - \frac{1}{1 + \pi_t} m_{t-1} - \tau_t \right) \right\},$$

where  $b_t$  is eliminated by use of (2), and where  $\mu_t$  is the Lagrange multiplier on (3). Interpret intuitively the first-order conditions for  $c_t$ ,  $n_t$ ,  $m_t$  and the expressions for the partial derivatives of  $V$ .

- (ii) Show that

$$i_t = \frac{\mu_{t+1}}{\beta V_b(m_{t+1}, b_{t+1})}, \quad (4)$$

and

$$-\frac{u_c(c_t, n_t)}{u_n(c_t, n_t)} = \frac{(\mu_t / [\beta V_b(m_t, b_t)]) + 1}{(1 - \alpha) A n_t^{-\alpha}}. \quad (5)$$

Discuss (4), and explain how nominal interest rate changes affect the labor supply decision (5).

- (iii) Consider the steady state. Apply the particular functional form of  $u$  and use (4)–(5) to derive the solution for employment as a function of the nominal interest rate, using that the national account is  $c_t = A n_t^{1-\alpha}$ . What is the optimal steady-state value of the nominal interest rate? [Hint: Use that optimal steady-state employment,  $n$ , solves  $\max_n \left\{ \log(A n^{1-\alpha}) - \frac{1}{1+\eta} n^{1+\eta} \right\}$ .] Explain.

**QUESTION 3:**

Consider the following New-Keynesian log-linear model of a closed economy:

$$y_t = E_t y_{t+1} - \sigma^{-1} \left( \hat{i}_t - E_t \pi_{t+1} \right), \quad \sigma > 0, \quad (1)$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa (y_t - y_t^n) \quad 0 < \beta < 1, \quad \kappa > 0, \quad (2)$$

$$\hat{i}_t = \phi \pi_t, \quad \phi > 1, \quad (3)$$

where  $y_t$  is output,  $\hat{i}_t$  is the nominal interest rate's deviation from steady state, and  $\pi_t$  is goods-price inflation,  $y_t^n$  is the natural rate of output, which is assumed to be a mean-zero i.i.d. shock.  $E_t$  is the rational-expectations operator conditional upon all information up to and including period  $t$ .

- (i) Derive the solutions for  $y_t$ ,  $\pi_t$  and  $\hat{i}_t$ . [Hint: Conjecture that the solutions are linear functions of  $y_t^n$ , and use the method of undetermined coefficients.] Explain how the shock is transmitted onto the variables.
- (ii) Assume that stabilizing the output gap,  $y_t - y_t^n$ , and  $\pi_t$  is preferable. Discuss the underlying model's welfare rationale for this assumption.
- (iii) Evaluate formally whether stabilizing  $y_t - y_t^n$  and  $\pi_t$  at the same time is possible in the model by appropriate choice of  $\phi$ . Discuss.